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# Asteroid “one-sided” families: Identifying footprints of YORP effect and estimating the age<sup>\*</sup>

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**Abstract.** In a previous paper (Icarus **274**, 314 (2016)) we have developed a new method to estimate the ages of asteroid families, based on an analysis of some expected consequence of the YORP effect acting on family members. The method is based on the identification of what we called the YORP eye, namely a region of depletion of the domain occupied by family members in the absolute magnitude - orbital semi-major axis plane. In this paper we develop a technique (“Mirroring tool”) to apply the same age determination method also in cases of strongly asymmetric families, for which it is difficult to assess the existence and location of a YORP eye. At the same time, we also apply some modifications to the procedure described in the previous paper. The results indicate that the mirroring tool can, in some cases, be satisfactorily effective, whereas in some other cases the results we obtain are more ambiguous. A necessary next step of this analysis will be to develop a better calibration of this method against the results obtained by means of other techniques, including primarily the age determination techniques based on analyses of the Yarkovsky-driven spreading in the semi-major axis of family members of different sizes.

## 1 Introduction

In a recent series of papers [1–5], we have published a new list of asteroid dynamical families<sup>1</sup>, for many of which we have been able to compute estimates of the age based on current knowledge of the size-dependent drift in the semi-major axis experienced by family members due to the Yarkovsky effect. Our technique, explained in these papers, makes use of the observed distribution of family members in the proper semi-major axis *vs.* size plane<sup>2</sup>. Since the smaller objects experience a faster drift in the semi-major axis due to the Yarkovsky effect, the domain occupied by family members in the semi-major axis *vs.* size plane takes a characteristic V-shaped morphology. The ages of families can be computed by an assessment of the aperture of the corresponding V-shaped domain [1, 3]. Throughout this paper, we will call such characteristic domains as “V-plots”, expressed in terms of proper semi-major axis (*a*) *vs.* absolute magnitude (*H*)<sup>3</sup>.

In another paper [6] (hereinafter referred to as Paper I) we performed a different analysis of the morphology of the V-plots. In particular, we looked for some possible footprints of the so-called YORP effect [7]. This is a complex mechanism producing a progressive change of both the spin rate and the orientation of the axis of rotation due to thermal emission effects from the irregular shape of an object. In particular, the interplay of the YORP and Yarkovsky effects tends to produce some depletion of objects in the central part of the V-plot of a family. In Paper I, we called “YORP eye”, such an underpopulated region of the V-plot, and we showed that its location, in terms of absolute magnitude, is time-dependent and can be used to derive an independent estimate of the age of a family.

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<sup>1</sup> A list of the families identified in our investigations, and of their members, is maintained at the AstDyS web site: <http://hamilton.dm.unipi.it/astdys/>.

<sup>2</sup> More precisely, we analyze plots of semi-major axis *vs.*  $1/D$ ,  $D$  being the size (in km) of family members computed from knowledge of the absolute magnitude and the average geometric albedo of the family.

<sup>3</sup> Needless to say, higher values of  $H$  correspond to smaller values of size.

In Paper I we computed such “YORP ages” for a number of big families, and we found a strong correlation with the ages of the same families derived by the more conventional method based on the pure Yarkovsky effect [3]. The main problem underlying this comparison is that indeed it is reasonable to assume that the time required for a given asteroid size to exhibit the maximum central depletion is correlated and approximately proportional to the typical timescale of a YORP cycle for the same size, but that at the moment we do not have any firm and reliable theoretical calibration. In Paper I we assumed that these two timescales were simply identical. With this assumption the “Yarkovsky” and “YORP” family ages were strongly correlated, but with a systematic offset. On average, the location of the “eye” expected on the basis of the “Yarkovsky” age was corresponding to smaller values of  $H$ . To obtain equal averages, we had to add a correction of 0.912 mag. However this calibration is empirical and strongly model-dependent. Even the moderate changes adopted in the present paper (see later) lead to a different (and probably larger) correction. This problem deserves further investigation that we plan to present in forthcoming papers, but we are no more taking care of it in the present one.

In what follows, we focus our attention on another problem that is encountered when carrying out analyzes of family data to identify the location of the YORP eye. The problem is that several families exhibit a strongly asymmetrical V-plot, making it difficult to find the location (or even to state the very existence) of a region of depletion at a value of  $a$  roughly corresponding to that of the family’s largest remnant. In other words, the two subsets of family members having  $a$  larger or lesser than the semi-major axis of the parent body ( $a_p$ ) can include very different numbers of family members, and in some cases one of the two subsets can even be totally absent.

There are several possible reasons for this. Some families were originated in cratering events which are intrinsically anisotropic, the fragments being possibly ejected along a preferential direction with respect to the parent body. Note, however, that an intrinsic anisotropy of the cratering process does not necessarily affect the distribution of the fragments in the semi-major axis (in some cases the asymmetry can in principle affect only the distribution in eccentricity or inclination).

In other cases (the best example being the family of (4) Vesta) the same parent body may have undergone more than one major cratering event at different epochs, with important consequences on the distribution of fragments originated from different events.

Another mechanism producing family asymmetry is possible in the vicinity of a dynamically unstable region, as in the case of a mean-motion resonance with Jupiter producing a Kirkwood gap. In such a case, the original family members injected by the collision into or close to the region of instability (so reaching some forbidden zone instantaneously or only later, due to a Yarkovsky-driven drift in semi-major axis  $a$ ) are removed immediately or over comparatively short (and size-dependent) timescales. When this happens, a family appears truncated and strongly asymmetric in semi-major axis.

The present paper is devoted to brief explanation of a method to deal with cases of strong family asymmetry, in order to be able to find evidence of the existence of YORP eyes and to determine their locations in the  $a$  vs.  $H$  plane.

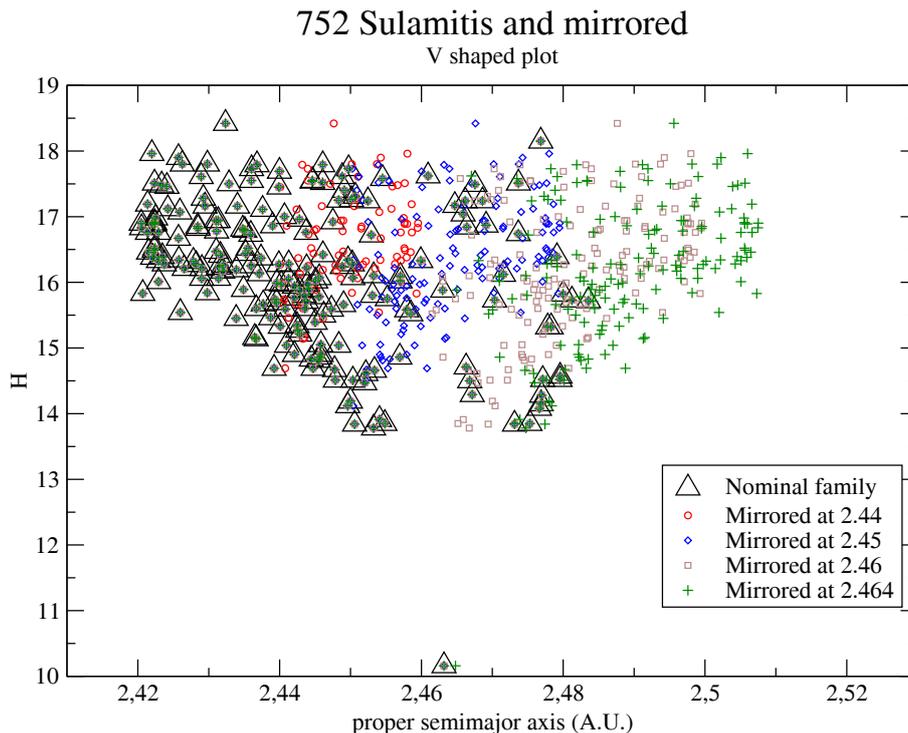
## 2 The mirroring tool

In the V-plot of an asymmetrical family, the proper semi-major axis of the largest remnant is usually strongly off-center, and thus different from the average (or the median)  $a$  value of the family. Hence, on one side the family appears to be less extended in  $a$  and includes a smaller number of members. As already noted above, these features can be due to different causes. In some cases the family could have been symmetrical at the epoch of its origin, and only the subsequent dynamical evolution could have produced the present asymmetry. It therefore makes sense to try to reconstruct the original properties using some plausible method. In what follows, we describe what we propose to use and what we call the *mirroring tool*.

The idea is rather simple. For any given family we define a few possible locations of an original symmetry axis, *i.e.* values of  $a$  for which the original V-plot is assumed to have been approximately symmetrical. Usually these values are very close to that of the largest remnant,  $a_p$ , entailing a “natural” mirroring, but we generally test also some other possible values. We call “wings” of the family the sets of family members having  $a$  smaller or larger than the chosen symmetry axis.

Having chosen a symmetry axis in  $a$ , a corresponding symmetric family (“mirrored family”) is then created in the following way:

- 1) The wing which contains the larger number of members is kept as it is.
- 2) New fictitious family members are added to the original members located in the opposite wing. The number of these synthetic members is equal to the difference between the population of the two wings, in such a way as to obtain a family with two equally populated wings.
- 3) A synthetic member is chosen randomly among the members of the more populated wing; to each of them we assign an identical value of  $H$  of the cloned real family member, and a value of  $a$  exactly symmetrical with respect to the chosen axis.



**Fig. 1.** The nominal family of (752) Sulamitis, and four mirrored families corresponding to different possible values of the adopted symmetry axis.

In this way we obtain a new family having a larger number of members, and exhibiting a much more symmetric shape. Note that only in extreme cases (perfectly one-winged families) we reach the limit of creating a fully symmetric family including a number of synthetic objects equal to the number of real family members.

In fig. 1 we show an example of application of the mirroring procedure. The family of (752) Sulamitis has been mirrored with respect to four possible symmetry axes. Note that the value  $a = 2.46$  au is only slightly lower than that of the largest body ( $a_p = 2.463$  au), while the value of  $a = 2.464$  au is larger, but also nearly coincident with  $a_p$ . As a consequence, the mirrored families corresponding to these two symmetry axes have very similar numbers of members (346 and 349, with respect to the number of real family members, namely 189). They show only a small offset in the  $a$  distribution of the synthetic objects (and, in the latter case, a duplicated largest body, something that in principle can be easily avoided by tuning the object generation software). It is easy to see that mirrored families, built as explained above and as shown in fig. 1, are by far more symmetric than the corresponding real families. Their properties will be analyzed in this paper in the search for footprints of the YORP effect (existence and location of a YORP eye in different cases). The purpose of this analysis is to understand whether and to which extent by using this simple mirroring trick we can overcome the problems posed by the dynamically driven evolution leading to strongly asymmetric families. In principle, all cratering families could have a significant asymmetry in the original relative velocity distribution, resulting in an asymmetry in the distribution of proper elements; however, this effect may or may not affect the distribution of semi-major axes, which is the one we are interested in.

Another example is represented in fig. 2. In this case the mirroring with a symmetry axis of  $a = 2.6374$  au closely corresponds to the largest member of the family, whereas other mirroring values correspond to more extreme choices.

### 3 Data analysis

In the present paper we have analyzed six asymmetrical families, namely those of (25) Phocaea, (145) Adeona, (752) Sulamitis, (945) Barcelona, (1658) Innes and (3827) Zdenekhorský. These families have been chosen because being affected to some extent by the presence of resonant and unstable regions.

Among them, (752) Sulamitis has a dominant (in size) largest remnant, and represents an example of a cratering event, whereas (1658) Innes and (3827) Zdenekhorský seem to be the outcomes of very energetic events producing extensive fragmentation. The others are intermediate cases. We were aware that our mirroring procedure may or may not work properly in the cases of cratering families (see the corresponding considerations above), while at the same time the identification of a symmetry axis is much more uncertain in the case of completely fragmented families.

We have already shown the cases of families (752) and (945). The asymmetric V-plots of the other four families included in our analysis are summarized in fig. 3.

### 945 Barcelona and mirrored

V shaped plot

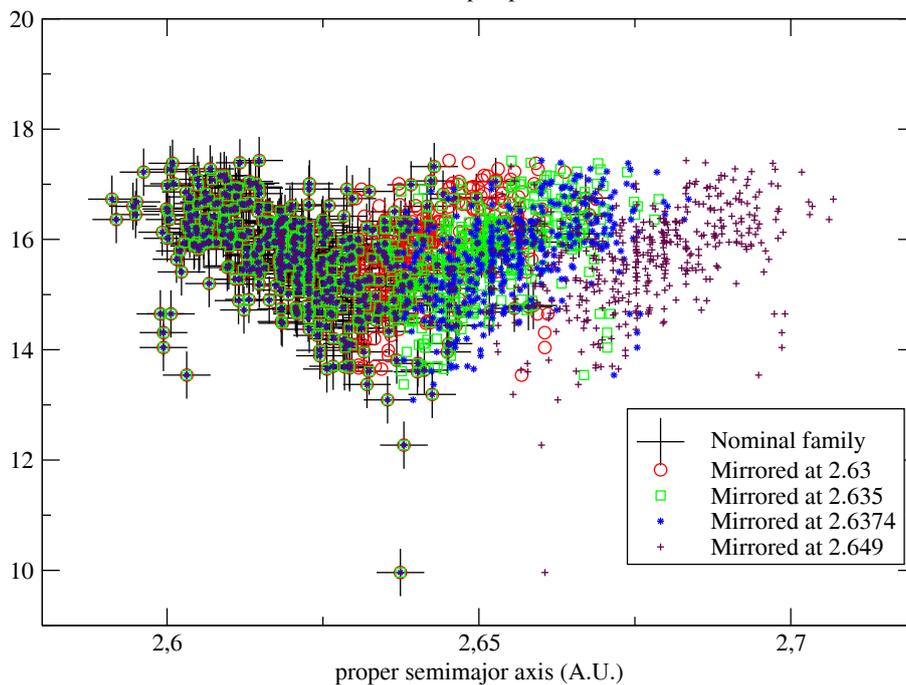
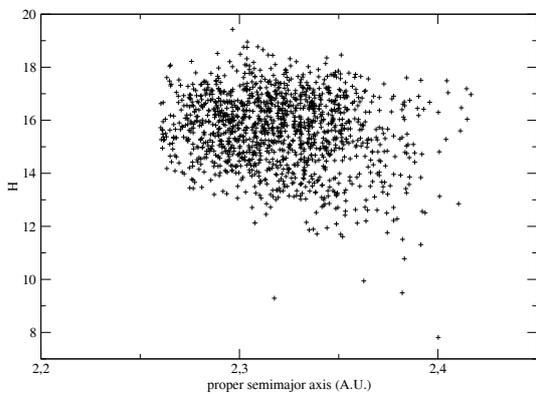
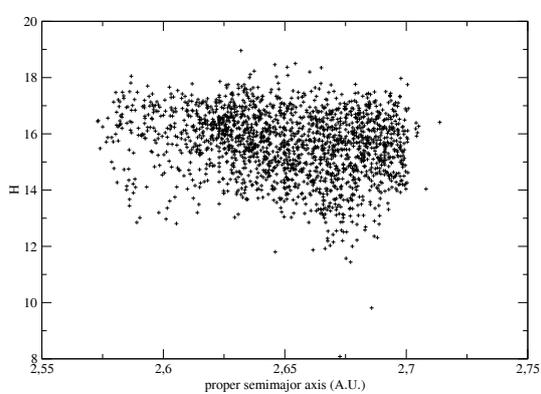


Fig. 2. The nominal family of (945) Barcelona and four mirrored families.

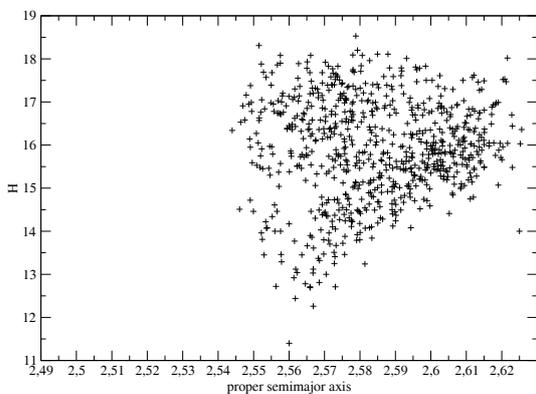
25 Phocaea - V shaped plot



145 Adeona - V shaped plot



1658 Innes - V shaped plot



3827 Zdenekhorský - V shaped plot

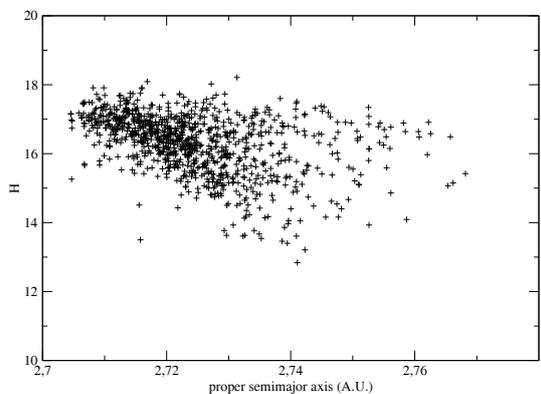


Fig. 3. V-plots of the nominal families of (25) Phocaea, (145) Adeona, (1658) Innes and (3827) Zdenekhorský.

### 3.1 Upgrades of the YORP-eye procedures

In this paper we used the new mirroring tool to overcome asymmetry problems in dealing with the six families of our sample. Apart from this, we used essentially the same data analysis methods described in Paper I. After some additional work and testing we have, however, introduced a few small modifications that we plan to apply also in the future analyzes of larger sample of families with sufficient number of members:

- We have redefined the  $H$  value of the computed maximum of the “central depletion parameter”  $R$  defined in Paper I, and used to evaluate in a quantitative way the presence or absence of a central depletion region due to the YORP effect (“YORP eye”) in the  $H$  vs.  $a$  plot for a given family. We divide the  $a$  range of the family, for a given range of  $H$ , into seven bins which are expected to contain the same number of bodies, if the value of  $a$  is completely due to the Yarkovsky effect and if the inclinations of the spin axis are isotropically distributed. The central depletion parameter is defined as  $R = \frac{3N_{\text{ext}}}{4N_{\text{int}}}$  where  $N_{\text{ext}}$  is the total population of the four external bins, and  $N_{\text{int}}$  that of the three internal bins. The value of  $R = 1$  corresponds to no central depletion, whereas increasingly higher values of  $R$  correspond to an increasingly stronger central depletion. Note that, according to the discussion in Paper I, the expected original value of  $R$ , before the action of Yarkovsky and YORP effects, is smaller than unity; thus also maxima with  $R < 1$  can be marginally interesting, and consistent with some evolution. The value of  $R$  is computed for subsets of family members having different values of  $H$ , the idea being that the location of a YORP eye in the V-plot of a family is time-dependent through the value of inverse size (equivalently,  $H$ ) at which the depletion is observed. More precisely, we compute  $R$  for subsets (that we call “boxes” in Paper I) of family members ordered in subsequent bins of  $H$ , from brightest to faintest values. Interested reader can find a more detailed explanation in Paper I. With respect to Paper I, we now associate to the  $R$  value of a given bin in  $H$  the average  $H$  value of the objects included in the considered box, and not any more the  $H$  value of the brightest family member included in the same box. According to this choice, the computed values of the  $H$  coordinate of the computed maximum value of  $R$  tend to be higher than the values found in Paper I.
- In general we are interested in extending our analysis to increasingly smaller families. In the present paper we already consider a family with less than 250 members, which was the lower limit for a reliable analysis adopted in Paper I. Thus, we have decided to decrease the size of the  $H$  boxes from 100 bodies used in Paper I, to 50 for most families in the present analysis, even down to 30 for families with a small number of members. Several, even if not all, tests have demonstrated that the results are not significantly affected by these changes.
- In Paper I we briefly discussed the equivalence of our adopted ( $R, H$ ) procedure to an alternative test based on the behavior of the Kurtosis of the distribution of  $a$  as function of  $H$ . We have, therefore, implemented in the code the computation of the momenta of the  $a$  distribution, including Skewness and Kurtosis [8]. In the present paper we are not performing a systematic comparison, but we rather show one example, confirming the good agreement between the results obtained using our original procedure and those that can be obtained from the analysis of the excess Kurtosis.

## 4 Results

We applied our mirroring procedure explained above to the six families of our sample. The details are summarized in table 1.

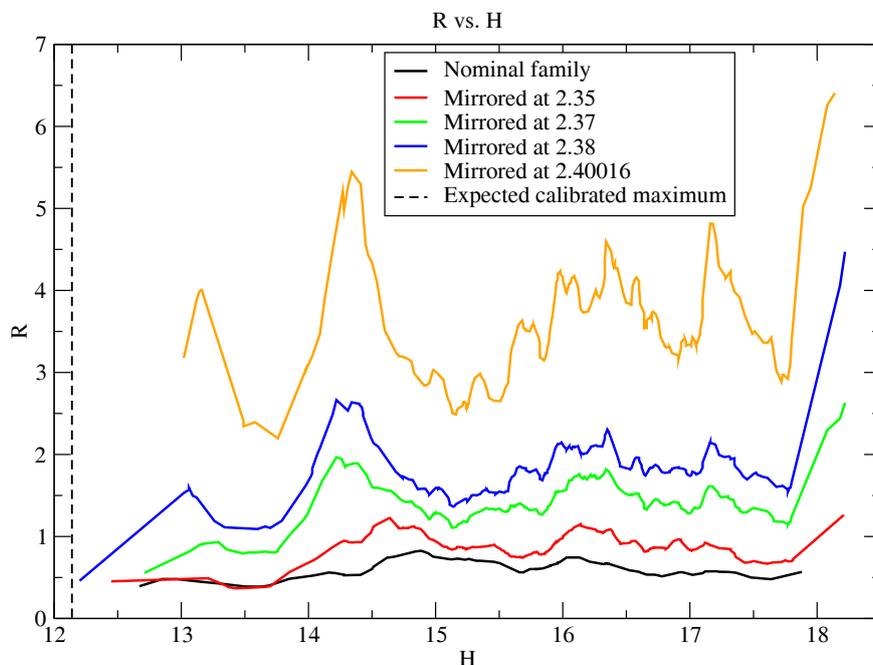
In figs. 4–9 we have plotted for each family the value of the central depletion parameter  $R$  as a function of  $H$ , and in each case we show the nominal and all the mirrored families. The vertical dashed line represents the expected location of the maximum, corresponding to the age computed by the Yarkovsky-based method, and under the assumptions discussed in Paper I (including the calibration of 0.912 mag in  $H$ ). Finally, as an example, we present, in fig. 10, the plot of the excess Kurtosis vs.  $H$  for the family 3827.

In analyzing the obtained results, we remark that, in general, the possible footprints (high values of  $R$  or, in the example with Kurtosis, low excess Kurtosis values) are most often found at larger values of  $H$  than expected. Such discrepancy can be, at least in part, a direct consequence of the new definition of the value of  $H$  assigned to each obtained value of the central depletion parameter  $R$ , as this may affect the computed  $H$  values for the maximum of  $R$ . The effect should be more pronounced in the case of older families, because for them the “YORP eye” is expected to be found in the region of less numerous large (and bright) objects. In fact, in this case, the difference between the  $H$  of the largest and the median body within a “box” is larger. To counterbalance this offset we need a revised calibration, but this is out of scope of the present paper and will be the subject of a forthcoming general analysis.

**Table 1.** The values “mirr*i*” ( $i = 1, 2, 3, 4$ ) are the chosen mirroring axes (in au), “ $N_i$ ” are the corresponding numbers of family members, and  $N_0$  gives the number of real family members. Note that for the family of (1658) Innes we have a set of decreasing mirroring axes, since this family is truncated at a value of  $a$  smaller than that of the largest remnant. “BOX” is the number of objects chosen to define the size of box used to compute the  $H$  value corresponding to the maximum of the central depletion parameter  $R$  (see text and Paper I).

| Family | $N_0$ | mirr1 | mirr2 | mirr3  | mirr4   | $N_1$ | $N_2$ | $N_3$ | $N_4$ | BOX |
|--------|-------|-------|-------|--------|---------|-------|-------|-------|-------|-----|
| 25     | 1405  | 2.35  | 2.37  | 2.38   | 2.40016 | 2590  | 2726  | 2759  | 2798  | 50  |
| 145    | 1946  | 2.63  | 2.65  | 2.66   | 2.685   | 2443  | 2855  | 3031  | 3606  | 50  |
| 752    | 189   | 2.44  | 2.45  | 2.46   | 2.464   | 270   | 322   | 346   | 349   | 30  |
| 945    | 428   | 2.63  | 2.635 | 2.6374 | 2.649   | 746   | 783   | 797   | 839   | 30  |
| 1658   | 751   | 2.58  | 2.57  | 2.565  | 2.56    | 1164  | 1323  | 1384  | 1426  | 50  |
| 3827   | 985   | 2.725 | 2.73  | 2.735  | 2.7412  | 1569  | 1716  | 1810  | 1889  | 50  |

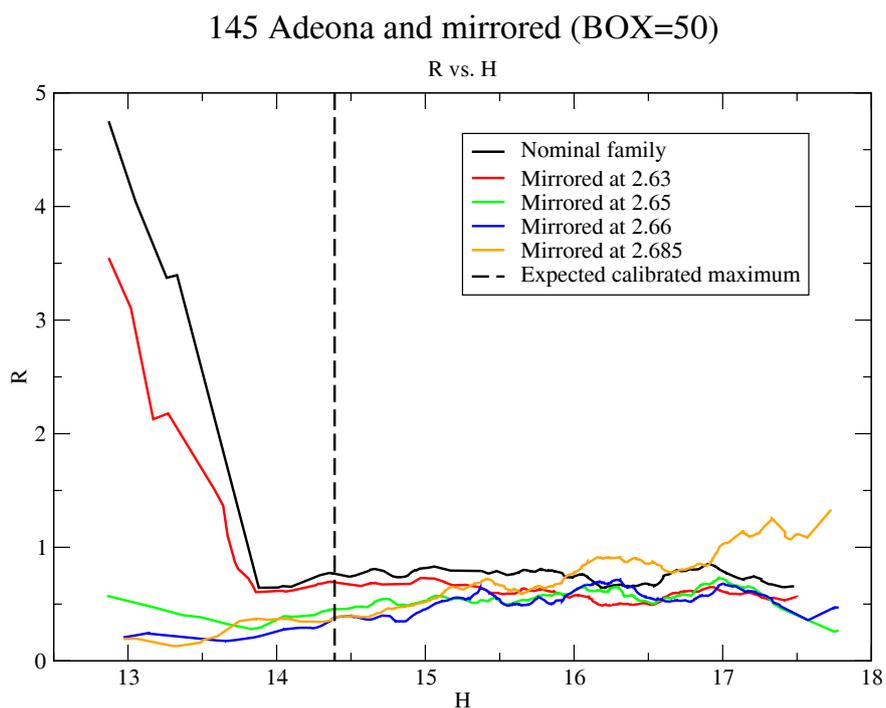
### 25 Phocaea and mirrored (BOX=50)



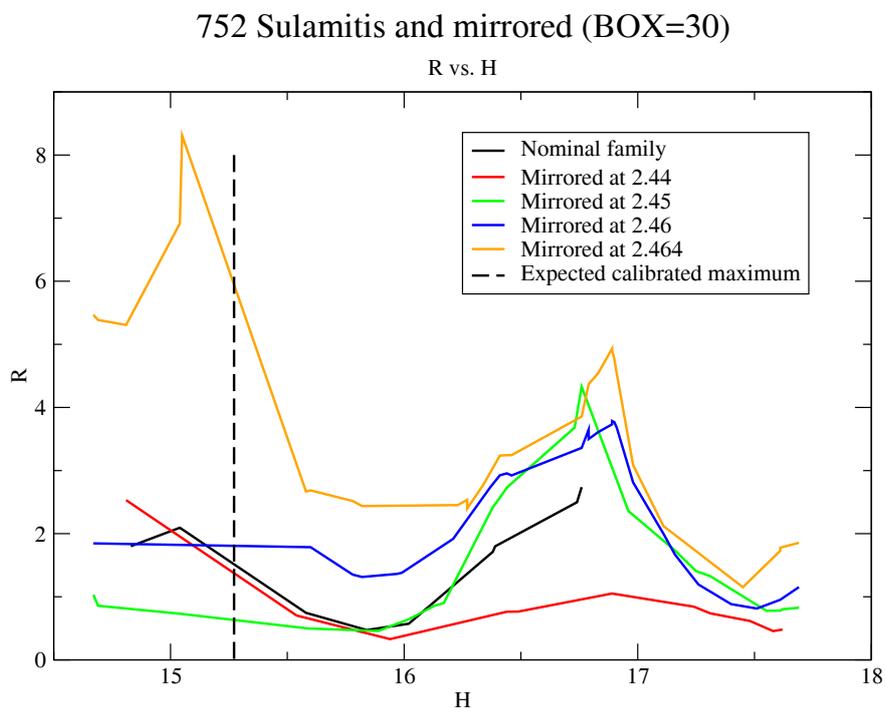
**Fig. 4.**  $R$  vs.  $H$  for the nominal family of (25) Phocaea and four mirrored synthetic families.

Apart from this point, we see that in general the mirroring process improves the results of the search. For instance, in the case of the family of (25) Phocaea (fig. 4), the nominal  $H$ - $R$  plot exhibits no significant maxima, and the value of  $R$  is always below unity (hence, more objects in the center than close to the borders). The mirroring procedure creates several significant peaks; for the family mirrored at 2.40016 au (orange curve) a very high peak ( $R \simeq 4$ ) appears at  $H \simeq 13$  mag, which is not far from the expected value.

The family of (752) Sulamitis (fig. 6) exhibits, in the extreme mirrored case, a very strong peak, but at lower  $H$  than the value expected on the basis of Yarkovsky-based results. The peak can be found in the plot for the nominal family too, but it is much less evident. We recall that this family is probably the outcome of a cratering event. Still, the mirroring tool seems to work, at least for what concerns the identification of the YORP eye. Indeed, if the family is analyzed looking for asymmetries in the distribution of the other orbital elements, enough asymmetry is found in the distribution of both proper  $e$  and proper  $\sin(I)$  to argue that it is not necessary to have an original asymmetry in the distribution of proper  $a$ . In general, the mirroring procedure we propose might fail with some cratering families, but can work with others, depending upon the direction in the proper elements space of the original anisotropy of the ejection velocities.



**Fig. 5.**  $R$  vs.  $H$  for the nominal family of (145) Adeona and four mirrored synthetic families.



**Fig. 6.**  $R$  vs.  $H$  for the nominal family of (752) Sulamitis and four mirrored synthetic families.

The nominal family of (945) Barcelona (fig. 7) exhibits its first significant peak at very high  $H$  value, while only the extreme mirrored family (orange) exhibits significant local maxima (*i.e.* with  $R$  larger than unity) for values of  $H$  not exceeding too much the expected one.

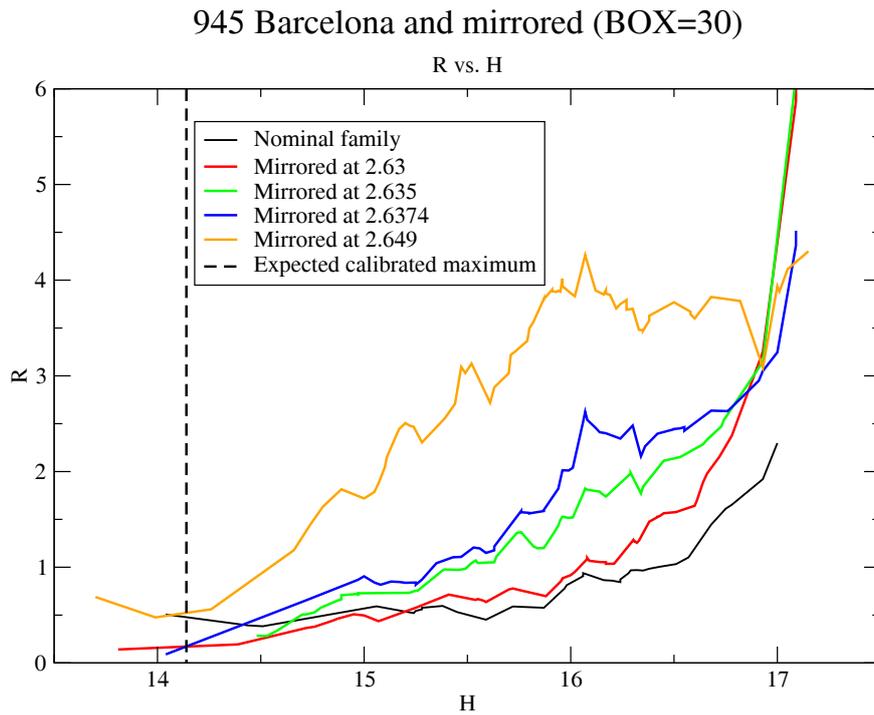


Fig. 7.  $R$  vs.  $H$  for the nominal family of (945) Barcelona and four mirrored synthetic families.

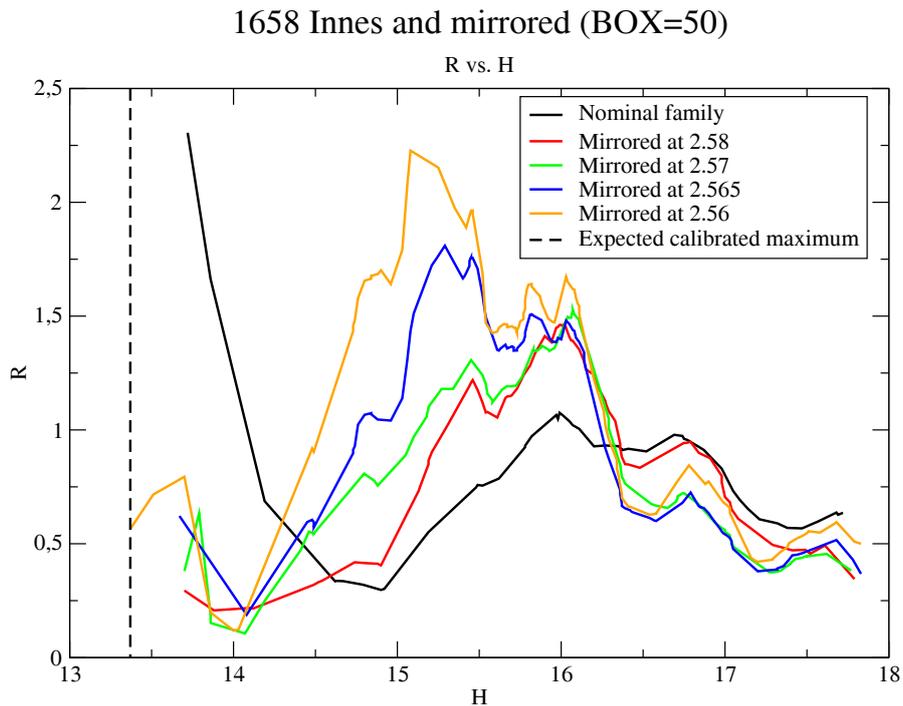


Fig. 8.  $R$  vs.  $H$  for the nominal family of (1658) Innes and four mirrored synthetic families.

The behavior of the family of (3827) Zdenekhorsky (fig. 9) is as expected. The extreme mirrored family exhibits a very strong  $R$  maximum very close to the expected  $H$ . A similar conclusion can be drawn also from the analysis of the Kurtosis plot (fig. 10): the strongly mirrored families have very low excess Kurtosis values around the region of  $H$  in which the “eye” is expected. Note that the excess Kurtosis must always be higher than  $-2$ , as this latter value corresponds to the situation in which all the bodies are at the edges of the distribution, thus a value such as  $-1.5$  can be considered as rather significant.

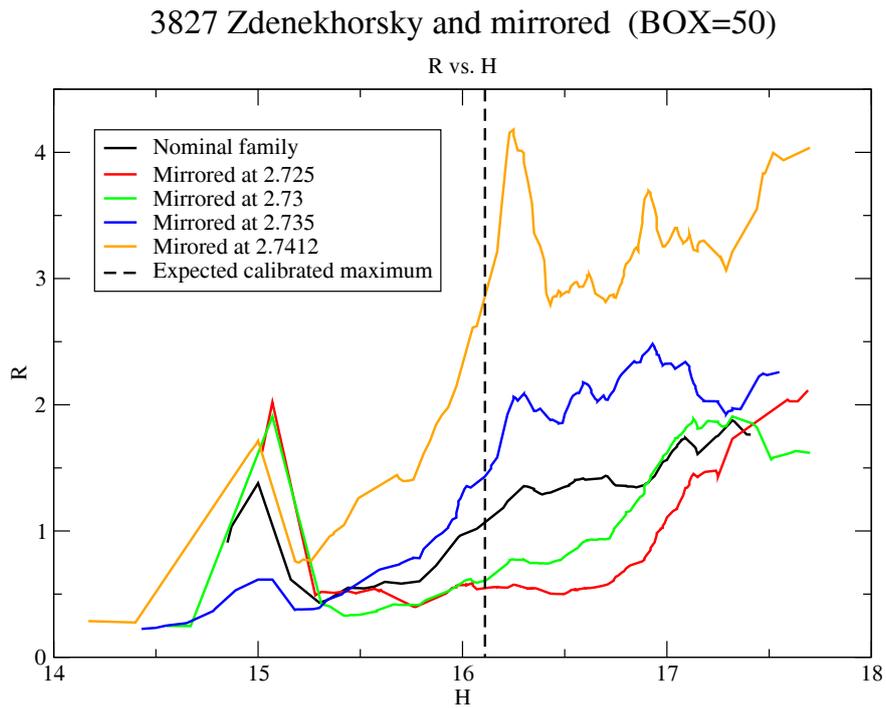


Fig. 9. *R vs. H* for the nominal family of (3827) Zdenekhorsky and four mirrored synthetic families.

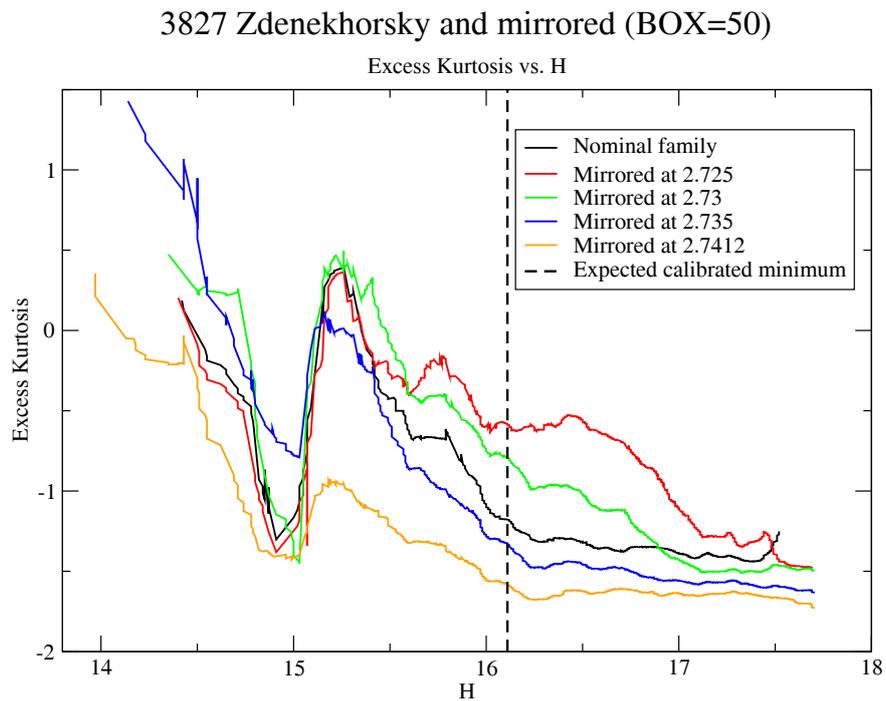


Fig. 10. Excess Kurtosis *vs. H* for the nominal family of (3827) Zdenekhorsky and four mirrored synthetic families.

#### 4.1 Dubious and unsatisfactory cases

In two cases the outcomes of the mirroring tool are ambiguous or unsatisfactory.

The behavior of the family of (1658) Innes (fig. 8) can be rated as ambiguous. The nominal family presents a maximum not far from the expected *H* value, while the mirrored ones keep a similar maximum, but with a lower value of *R*; moreover, they exhibit a new, strong, possibly significant maximum at higher *H* value, with an (acceptable) excess in *H* of about 1.5 mag. A more thorough analysis of this case is certainly needed.

The only completely unsatisfactory case is that of the family of (145) Adeona (fig. 5). Only the nominal family and a moderately mirrored version present a significant  $R$  maximum. All other cases present a flat and always low  $H$ - $R$  plot. However, the dynamical processes affecting Adeona family are rather complex, and this negative result is not entirely surprising.

In [5] (sect. 4.1) it is argued that the family of (145) Adeona is not just one sided, in that the 8/3 mean motion resonance with Jupiter has essentially wiped out half of the original family at larger semi-major axes  $a$ , but that it is also affected by rather complex dynamical and/or collisional effects also in the other, low  $a$  half.

One possible interpretation, see [5] (fig. 18) is that three mean motion resonances (involving both Jupiter and Saturn) located around  $a = 2.62$  au have depleted the family region from most asteroids moving towards lower values of  $a$ , thus the membership is strongly affected and this destroys any possible information on the YORP eye. Another possible interpretation is that the significant number of outliers found for  $a < 2.62$  au in the fit for the V-shape used to compute the family age, see [5] (fig. 18) actually belong to a separate collisional family. The same paper advocates the need of observational tests to decide between these two interpretations.

Whatever the chosen interpretation of the Adeona family, it is obvious that under these conditions a mirror procedure could not succeed in reconstructing the YORP history of the family. We propose that the family 145 is to be considered as an example of the fact that the mirroring procedure we have developed does not work always, thus the fact that it either does or does not provide an answer on the presence of a detectable YORP eye is significant.

## 4.2 Conclusions

The present preliminary analysis of a few asymmetrical families shows that the mirroring tool may be helpful in improving the identification of the YORP effect footprints, but the method certainly has to be tested on a case-by-case basis considering a larger sample of families, and possibly after certain improvements we expect from the future, including independent theoretical considerations (relevance of dynamical processes, other possible causes of asymmetry).

Moreover, the preliminary analysis presented in this paper certainly shows that the calibration of our method is still essentially arbitrary, and can be obtained only *a posteriori*, from the comparison of the expected and computed maxima. Apart from this comparison, a theoretical analysis of the YORP-Yarkovsky-driven evolution, capable of taking into account the presence of possible correlations (for instance, spin-ejection velocity) among the original properties, and then collisional disruption or reshaping, collisional spin reorientation and so on, is certainly needed.

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## References

1. A. Milani, A. Cellino, Z. Knežević, B. Novaković, F. Spoto, P. Paolicchi, *Icarus* **239**, 46 (2014).
2. Z. Knežević, A. Milani, A. Cellino, B. Novaković, F. Spoto, P. Paolicchi, *Complex Planetary Systems* (Cambridge University Press, 2014) pp. 130–133.
3. F. Spoto, A. Milani, Z. Knežević, *Icarus* **257**, 275 (2015).
4. A. Milani, F. Spoto, Z. Knežević, B. Novaković, G. Tsirvoulis, *Asteroids: New Observations New Models* (Cambridge University Press, 2016) pp. 28–45.
5. A. Milani, Z. Knežević, F. Spoto, A. Cellino, B. Novaković, G. Tsirvoulis, *Icarus* **288**, 240 (2017).
6. P. Paolicchi, Z. Knežević, *Icarus* **274**, 314 (2016).
7. D.P. Rubincam, *Icarus* **148**, 2 (2000).
8. O.J. Dunn, V.A. Clark, *Applied Statistics: Analysis of Variance and Regression* (Wiley, New York, 1974) pp. 610–613.